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Calculation of Wave Shoaling With Dissipation Over Nearshore Sands

by
Robert J. Hallermeier

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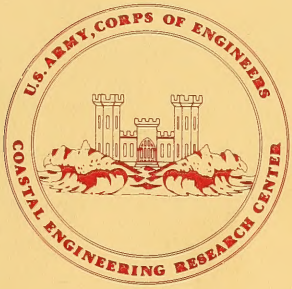

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This report provides a simplified calculation procedure for nearshore wave height changes considering the energy dissipated by rough turbulent flow over a strongly agitated bed of quartz sand. All elementary wave relationships are from linear monochromatic wave theory, but one effect of including dissipation is that calculated height changes depend on the absolute wave height. The general effect of appreciable energy loss is to make field wave height relatively constant outside the breaker zone. Example computations and a calculator program are provided.

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PREFACE

This report provides a calculation procedure for nearshore shoaling of energetic waves outside the breaker zone, including the appreciable effects of energy dissipation over a strongly agitated sand bed. The work reported was conducted under the U.S. Army Coastal Engineering Research Center's (CERC) Numerical Modeling of Shoreline Response to Coastal Structures work unit, Shore Protection and Restoration Research Program, Coastal Engineering Area of Civil Works Research and Development.

The present treatment replaces guidance previously provided in CERC Field Guidance Letter No. 79-04 and CERC Technical Paper No. 80-8 (Grosskopf, 1980). Those publications incorrectly recommend the use of friction coefficients for inert beds in computing nearshore wave shoaling.

The report was written by Dr. Robert J. Hallermeier, Oceanographer, under the general supervision of Mr. R.P. Savage, Chief, Research Division, CERC.

Technical Director of CERC was Dr. Robert W. Whalin, P.E.

Comments on this publication are invited.

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TED E. BISHOP
Colonel, Corps of Engineers
Commander and Director

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CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	by	To obtain
inches	25.4	millimeters
	2.54	centimeters
square inches	6.452	square centimeters
cubic inches	16.39	cubic centimeters
feet	30.48	centimeters
	0.3048	meters
square feet	0.0929	square meters
cubic feet	0.0283	cubic meters
yards	0.9144	meters
square yards	0.836	square meters
cubic yards	0.7646	cubic meters
miles	1.6093	kilometers
square miles	259.0	hectares
knots	1.852	kilometers per hour
acres	0.4047	hectares
foot-pounds	1.3558	newton meters
millibars	1.0197×10^{-3}	kilograms per square centimeter
ounces	28.35	grams
pounds	453.6	grams
	0.4536	kilograms
ton, long	1.0160	metric tons
ton, short	0.9072	metric tons
degrees (angle)	0.01745	radians
Fahrenheit degrees	5/9	Celsius degrees or Kelvins[1]

[1]To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use formula: $C = (5/9)(F - 32)$.

To obtain Kelvin (K) readings, use formula: $K = (5/9)(F - 32) + 273.15$.

SYMBOLS AND DEFINITIONS

$c = L/T$	wave celerity
D	median sand grain diameter
d	mean water depth
d_a	maximum water depth for sand bed agitation by waves
$\overline{E}$	average energy dissipation rate
f_e	energy dissipation coefficient for rough turbulent flow over strongly agitated sand bed
g	acceleration due to gravity
H	wave height
H_o'	equivalent wave height in deep water ignoring refraction
K_s	shoaling coefficient in linear wave theory
L	wavelength
$L_o = gT^2/2\pi$	wavelength in deep water
n	ratio of group velocity to wave celerity
$\overline{P}$	average wave energy flux
T	wave period
X	wave propagation distance
ξ	horizontal amplitude of near-bed fluid orbit
ρ	fluid density

Additional subscripts

j	value at location where wave condition is to be predicted
m	value at geometric mean water depth for region of interest
1	value at location where wave condition is defined initially
$+/-$	case where d_j is greater or less than d_1

CALCULATION OF WAVE SHOALING WITH DISSIPATION OVER NEARSHORE SANDS

by

Robert J. Hallermeier

I. INTRODUCTION

As waves propagate toward breaking in shallow water, their attributes are transformed by effects of water depth and bottom features. These nearshore transformations are crucial in the interpretation of wave measurements and in the prediction of sediment transport. Prior to wave breaking, appreciable wave energy can be dissipated by friction between the oscillatory water motion and the nearshore bottom, especially where waves cause strong agitation of bottom sediments. This report considers such frictional dissipation.

Ocean waves can be represented most adequately as distributions of propagating energy with respect to frequency and direction. Near the shore, extremely energetic waves are observed to constitute a somewhat regular wave train approaching along a shore-normal line, with a rather well-defined wave height and period. According to the Shore Protection Manual (SPM) (U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1977), an idealization of demonstrated value in coastal engineering is to represent real waves by a characteristic height and period, the significant wave condition; this wave representation is utilized here.

This report provides a simple calculation procedure defining changes in significant wave height due to water depth differences combined with bottom friction at agitated sand beds. The procedure uses linear wave theory between the two depths of interest and a computation of energy dissipation rate at an intermediate water depth. Factors in wave transformation ignored here include: wave direction and complex spectra, currents, surface wave breaking, winds, water viscosity, and bottom percolation and elasticity. The technique is meant for application only to energetic field wave conditions in fairly shallow water with straight, parallel depth contours and relatively fine quartz bottom sands.

Section II presents the calculation procedure for wave height changes considering energy dissipation. Section III addresses the application of this procedure and includes two example problems: converting nearshore wave measurements into equivalent wave heights in shallower and in deeper water. The reader is referred to Hallermeier (in preparation, 1983) for a detailed substantiation of elements incorporated in the calculation procedure; that reference reviews the empirical basis for the expression giving energy dissipation coefficient and reports extensive calculated results in clear agreement with multiple wave measurements at the Coastal Engineering Research Center's (CERC) Field Research Facility, Duck, North Carolina, and at other sites.

II. CALCULATION PROCEDURE

Waves are presumed to travel perpendicular to depth contours with propagation described by small-amplitude (linear) wave theory. Required theoretical relations are provided in Section 2.23 of the SPM. Wave and fluid characteristics arising in the wave description are

d = local mean water depth

g = acceleration due to gravity

H = local wave height

L = local wavelength

T = wave period

ρ = fluid density

Wavelength in deep water is $L_o = (gT^2/2\pi)$ and the dimensionless local wavelength, d/L, is the solution of

$$\frac{d}{L} \tanh\left(\frac{2\pi d}{L}\right) = \frac{d}{L_o} \tag{1}$$

which is presented in Table C-1 of the SPM. The average wave energy flux per unit crest width is

$$\overline{P} = \frac{1}{8} \rho \, g \, H^2 \, c \, n \tag{2}$$

where $c = (L/T)$ is wave celerity and n the ratio of group velocity to wave celerity:

$$n = \frac{1}{2}\left[1 + \frac{4\pi d/L}{\sinh\,(4\pi d/L)}\right] \tag{3}$$

With no wave refraction, and provided no energy has been added to or removed from the wave train, it is convenient to express wave height changes by the factor

$$\frac{H}{H_o'} = \frac{1}{\sqrt{2n \tanh\,(2\pi d/L)}} = K_S \tag{4}$$

where H_o' is equivalent wave height in deep water (ignoring refraction), and K_S the shoaling coefficient. The dimensionless quantities n and K_S are provided for specific values of (d/L_o) in Table C-1 of the SPM. In situations considered here, a nearshore wave measurement at water depth d_1 is to be converted into the corresponding wave condition at another relatively shallow depth, d_j. The wave period is presumed constant during propagation so that (d_1/L_o) and (d_j/L_o) are known, and equation (4) would give the ratio of wave heights as

$$\frac{H_j}{H_1} = \frac{K_{sj}}{K_{s1}} \tag{5}$$

if energy dissipation were to be ignored.

Energy lost from the wave train due to bottom friction is treated by means of a single dissipation calculation at the geometric mean depth for the region of interest

$$d_m = \sqrt{d_1 d_j} \tag{6}$$

Average energy dissipation rate at d_m, per unit crest width and per unit length in the propagation direction, is given by

$$\overline{E}_m = 0.235\ \rho\ f_{em}\ (2\pi\ \xi_m/T)^3 \tag{7}$$

For rough turbulent flow over a strongly agitated bed of quartz sand, the energy dissipation coefficient introduced in equation (7) is

$$f_{em} = \exp\left[-5.882 + 14.57\ (D_m/\xi_m)^{0.194}\right] \tag{8}$$

Here D_m is median sand grain diameter at d_m and ξ_m is horizontal amplitude of the near-bed fluid excursion arising at d_m without energy dissipation, so that $(2\pi\xi_m/T)$ in equation (7) is peak near-bed fluid velocity. According to linear wave theory,

$$\xi_m = \frac{H_m}{2\ \sinh\ (2\pi d_m/L_m)} \tag{9}$$

where $H_m = (H_1\ K_{sm}/K_{s1})$ from equation (5).

Energy-conserving linear wave shoaling is combined with computed energy dissipation rate into an expression giving a wave height at d_j equivalent to measured wave height at d_1. This expression is a revised form of equation (2):

$$H_{j\pm}^2 = \frac{8(\overline{P}_1 \pm \overline{E}_m X)}{\rho\ g\ c_j\ n_j} \tag{10}$$

where X is the wave propagation distance between the two water depths (d_1 and d_j), and the upper [lower] sign is used when d_j is greater [less] than d_1. The conversion given in equation (10) presumes that computed dissipation rate at d_m can be considered representative of the entire propagation path.

III. APPLICATIONS

Besides the explicitly ignored factors affecting nearshore wave transformations, it is important in applications to consider the requirements stated above on the use of equation (8) for energy dissipation coefficient. The quartz sand bed must be strongly agitated by wave action and near-bed flow must be rough turbulent. Appropriate situations correspond to nearshore field waves with relatively large height and period.

The strength of bed agitation may be judged using an approximate expression (Hallermeier, 1981, eq. 10) giving maximum water depth, d_a, for wave agitation

of a quartz sand bed when viscous effects are negligible:

$$d_a = HT\,(g/5000\,D)^{0.5} \qquad (11)$$

If d_a computed from H_1, T_1, and D_m is much larger than the maximum water depth of interest, the requirement for strong bed agitation may be considered satisfied.

To assess whether flow is likely to be rough turbulent, another simple computation can be performed. Incorporating the same approximation for ξ in intermediate water depth ($2\pi d/L$ near unity) as is utilized in equation (11), fundamental results reviewed in Hallermeier (in preparation, 1983) support

$$HT \geq d \qquad \{\text{metric units}\} \qquad (12)$$

as an approximate criterion for rough turbulent flow at a strongly agitated bed of quartz sand. If equation (12) is true according to H_1, T_1, and the maximum water depth of interest, the requirement for rough turbulent near-bed flow may be considered satisfied.

The following example problems demonstrate the use of the present procedure in calculating nearshore wave shoaling with energy dissipation due to a strongly agitated sand bed.

* * * * * * * * * * * * * * EXAMPLE PROBLEM 1 * * * * * * * * * * * * * *

GIVEN: At the CERC Field Research Facility, significant wave height exceeding 3.5 meters was recorded during three 1981 storms by a Waverider buoy located in an 18-meter mean water depth. Wave periods associated with these extremely high waves ranged from 9.3 to 14.0 seconds. Nearshore bathymetry at this site is regularly surveyed to the 9-meter water depth contour, and the wave characteristics at the seaward boundary to the survey region are of interest. The shore-normal distance between the water depths is 1800 meters, and the representative sand size for the intervening bottom is $D = 0.12$ millimeter.

FIND: The wave height at $d_j = 9$ meters corresponding to $H_1 = 3.5$ meters at $d_1 = 18$ meters for: (a) $T_1 = 9.3$ seconds and (b) $T_1 = 14.0$ seconds.

SOLUTION:

(a) For $T_1 = 9.3$ seconds and $d_1 = 18$ meters

$$\frac{d_1}{L_o} = \frac{2\pi d_1}{gT_1^2} = \frac{(2\pi)(18)}{9.81\,(9.3)^2} = 0.1333$$

Table C-1 in the SPM gives $n_1 = 0.7570$, $(H_1/H_o') = 0.9160 = K_{s1}$ and $d_1/L_1 = 0.1694$, so that $L_1 = 106.3$ meters. With $H_1 = 3.5$ meters and $\rho = 1026$ kilograms per cubic meter for saltwater, equation (2) becomes

$$\bar{P}_1 = \frac{1}{8} \rho g H_1^2 c_1 n_1 = \frac{1}{8} (1026)(9.81)(3.5)^2 \frac{106.3}{9.3} (0.757)$$

$$= 1.33 \cdot 10^5 \text{ kilogram-meter per second cubed}$$

Since $d_m = \sqrt{d_1 d_j} = \sqrt{(9)(18)} = 12.73$ meters

$$\frac{d_m}{L_o} = \frac{(2\pi)(12.73)}{9.81\ (9.3)^2} = 0.09427$$

and Table C-1 gives $(d_m/L_m) = 0.1360$, $\sinh(2\pi d_m/L_m) = 0.9621$, $(H_m/H_o') = 0.9378 = K_{sm}$, and $n_m = 0.8199$. Thus, $H_m = (H_1 K_{sm}/K_{s1}) = (3.5)(0.9378)/(0.9160) = 3.58$ meters and according to equation (9),

$$\xi_m = \frac{H_m}{2 \sinh\left(\frac{2\pi d_m}{L_m}\right)} = \frac{3.58}{2(0.9621)} = 1.86 \text{ meters}$$

With $D_m = 0.12$ millimeter, equation (8) is

$$f_{em} = \exp\left[-5.882 + 14.57 (D_m/\xi_m)^{0.194}\right]$$

$$= \exp\left[-5.882 + 14.57 (1.2 \cdot 10^{-4}/1.86)^{0.194}\right] = 0.0262$$

so that equation (7) becomes

$$\bar{E}_m = 0.235 \rho f_{em}\left(\frac{2\pi\xi_m}{T}\right)^3 = (0.235)(1026)(0.0262)\left[\frac{2\pi(1.86)}{9.3}\right]^3$$

$$= 12.5 \text{ kilograms per second cubed}$$

Some conditions must also be computed at $d_j = 9$ meters, where

$$\frac{d_j}{L_o} = \frac{(2\pi)\ 9}{(9.81)(9.3)^2} = 0.06665$$

so that Table C-1 gives $(H_j/H_o') = 0.9779 = K_{sj}$, $n_j = 0.8688$, and $(d_j/L_j) = 0.1108$ so that $L_j = 81.2$ meters. Finally, because $d_j < d_1$ the lower sign in equation (10) is appropriate, and $X = 1800$ meters yields

$$H_j^2 = \frac{8(\bar{P}_1 - \bar{E}_m X)}{\rho g c_j n_j} = \frac{8[1.33 \cdot 10^5 - (12.5)(1800)]}{(1026)(9.81)\ \frac{(81.2)}{9.3}\ (0.8688)} = 11.58 \text{ square meters}$$

$$H_j = 3.40 \text{ meters}$$

From equation (11), maximum water depth for bed agitation is $d_a = H_1 T_1 (g/5000 D)^{0.5} = (3.5)(9.3)[9.81/(5 \cdot 10^3)(0.12 \cdot 10^{-3})]^{0.5} = 131.6$ meters,

much larger than water depths in the region treated, and the numerical value in metric units of $(H_1 T_1) = 32.6$, nearly twice the maximum water depth considered in meters, so that equation (12) indicates rough turbulent flow throughout the region. The calculation procedure is suitable for these conditions, and the effect of bottom friction on wave shoaling is appreciable, in that linear wave theory without dissipation would predict a nearshore wave height of $(H_1 K_{sj}/K_{s1}) = [(3.5)(0.9779)/0.9160] = 3.74$ meters, using equation (5).

(b) For $T_1 = 14.0$ seconds and $d_1 = 18$ meters, $(d_1/L_o) = 0.05882$ so that $n_1 = 0.8833$, $(H_1/H_o') = 0.9963 = K_{s1}$, $(d_1/L_1) = 0.1031$ and $L_1 = 174.6$ meters from Table C-1. Equation (2) with $H_1 = 3.5$ meters gives

$$\overline{P}_1 = \frac{1}{8}(1026)(9.81)(3.5)^2 \frac{(174.6)}{(14.0)}(0.8833)$$

$$= 1.70 \cdot 10^5 \text{ kilogram-meter per second cubed}$$

At d_m, $(d_m/L_o) = 0.04160$ so that $(d_m/L_m) = 0.08509$, $\sinh(2\pi d_m/L_m) = 0.5605$, $(H_m/H_o') = 1.057 = K_{sm}$, and $n = 0.9161$. Thus, $H_m = (H_1 K_{sm}/K_{s1}) = 3.71$ meters, equation (9) is

$$\xi_m = \frac{3.71}{2(0.5605)} = 3.31 \text{ meters}$$

equation (8) is

$$f_{em} = \exp\{-5.882 + 14.57 (1.2 \cdot 10^{-4}/3.31)^{0.194}\} = 0.0207$$

and equation (7) is

$$\overline{E}_m = (0.235)(1026)(0.0207)\left[\frac{2\pi(3.31)}{14.0}\right]^3$$

$$= 16.36 \text{ kilograms per second cubed}$$

At d_j, $(d_j/L_o) = 0.02941$ so that $(H_j/H_o') = 1.130 = K_{sj}$, $n_j = 0.9400$, $(d_j/L_j) = 0.0706$ and $L_j = 127.5$ meters. Again, the lower sign in equation (10) is used to give

$$H_j^2 = \frac{8[1.70 \cdot 10^5 - (16.36)(1800)]}{(1026)(9.81)\frac{(127.5)}{(14.0)}(0.9400)} = 13.05 \text{ square meters}$$

$$H_j = 3.61 \text{ meters}$$

Because T is greater here than in part (a), the requirements for rough turbulent flow at a strongly agitated sand bed are clearly satisfied. Although the computation including friction results in H_j slightly larger than H_1 in this case, energy dissipation again has an appreciable effect since linear wave theory would predict a nearshore wave height of $(H_1 K_{sj}/K_{s1}) = 3.97$ meters, from equation (5).

* * * * * * * * * * * * * * * EXAMPLE PROBLEM 2 * * * * * * * * * * * * * * *

GIVEN: A mathematical model is to be used to simulate storm wave effects for water depths shoreward of 9 meters at Nags Head, North Carolina, with the threshold for storm waves taken to be the wave height exceeded 10 percent of the time. The wave climate at this site has been defined by several relatively complete years of data from a pier-mounted gage located in mean water depth of 5.2 meters (Thompson, 1977): wave height exceeded in 10 percent of these measurements is about 1.7 meters and the typical wave period for this wave height is about 8.5 seconds. The shore-normal distance between 5.2- and 9-meter water depth is 600 meters; representative sand size for the intervening bottom is $D = 0.20$ millimeter.

FIND: The wave height at $d_j = 9$ meters corresponding to $H_1 = 1.7$ meters and $T_1 = 8.5$ seconds at $d_1 = 5.2$ meters.

SOLUTION: For $T_1 = 8.5$ seconds and $d_1 = 5.2$ meters,

$$\frac{d_1}{L_o} = \frac{2\pi d_1}{gT_1^2} = \frac{(2\pi)(5.2)}{(9.81)(8.5)^2} = 0.04610$$

Table C-1 in SPM gives $n_1 = 0.9074$, $(H_1/H_o') = 1.038 = K_{s1}$ and $d_1/L_1 = 0.09002$, so that $L_1 = 57.76$ meters. With $H_1 = 1.7$ meters, equation (2) is

$$\overline{P}_1 = \frac{1}{8} \rho g H_1^2 c_1 n_1 = \frac{1}{8} (1026)(9.81)(1.7)^2 \frac{(57.76)}{(8.5)} (0.9074)$$

$$= 2.24 \cdot 10^4 \text{ kilogram-meter per second cubed}$$

Since $d_m = \sqrt{d_1 d_j} = \sqrt{(5.2)(9)} = 6.84$ meters

$$\frac{d_m}{L_o} = \frac{(2\pi)(6.84)}{(9.81)(8.5)^2} = 0.06064$$

and Table C-1 gives $(d_m/L_m) = 0.1049$, $\sinh(2\pi d_m/L_m) = 0.7082$, $(H_m/H_o') = 0.9916 = K_{sm}$, and $n_m = 0.8799$. Thus, $H_m = (H_1 K_{sm}/K_{s1}) = (1.7)(0.9916)/1.038 = 1.624$ meters and according to equation (9),

$$\xi_m = \frac{H_m}{2 \sinh\left(\frac{2\pi d_m}{L_m}\right)} = \frac{1.624}{2(0.7082)} = 1.15 \text{ meters}$$

With $D_m = 0.20$ millimeter, equation (8) is

$$f_{em} = \exp\{-5.882 + 14.57 (D_m/\xi_m)^{0.194}\}$$

$$= \exp\{-5.882 + 14.57 [2 \cdot 10^{-4}/1.15]^{0.194}\} = 0.0422$$

so that equation (7) is

$$\bar{E}_m = 0.235\ \rho\ f_{em}\left(\frac{2\pi\xi_m}{T}\right)^3 = (0.235)(1026)(0.0422)\left[\frac{2\pi\ (1.15)}{8.5}\right]^3$$

$$= 6.25 \text{ kilograms per second cubed}$$

At $d_j = 9$ meters,

$$\frac{d_j}{L_o} = \frac{(2\pi)(9)}{(9.81)(8.5)^2} = 0.0798$$

so that Table C-1 gives $(H_j/H'_o) = 0.9551 = K_{sj}$, $n_j = 0.845$, and $(d_j/L_j) = 0.1230$ so that $L_j = 73.2$ meters. Finally, because $d_j > d_1$ the upper sign in equation (10) is appropriate, and $X = 600$ meters yields

$$H_j^2 = \frac{8\ (\bar{P}_1 + \bar{E}_m X)}{\rho\ g\ c_j\ n_j} = \frac{8\ [2.24 \cdot 10^4 + (6.25)(600)]}{(1026)(9.81)\ \frac{(73.2)}{(8.5)}\ (0.845)} = 2.86 \text{ square meters}$$

$$H_j = 1.69 \text{ meters}$$

To show that the calculation procedure is suitable for these conditions, maximum water depth for bed agitation from equation (11) is

$$d_a = H_1 T_1 \left[\frac{g}{5000\ D}\right]^{0.5} = (1.7)(8.5)\left[\frac{9.81}{(5 \cdot 10^3)(2 \cdot 10^{-4})}\right]^{0.5}$$

$$= 45.3 \text{ meters}$$

much larger than water depths in the region treated, and the numerical value in metric units of $(H_1 T_1) = 14.45$, greater than the maximum water depth considered in meters, so that equation (12) indicates rough turbulent flow throughout the region considered. The effect of bottom friction is still appreciable for this relatively low-energy case, in that linear wave theory without dissipation would provide a wave height according to equation (5) of $(H_1\ K_{sj}/K_{s1}) = [(1.7)(0.9551)/1.038] = 1.56$ meters at 9-meter water depth.

* *

With linear wave theory, the height change between two water depths depends on wave period. Although only linear theory wave relationships are incorporated in the present calculation procedure, energy dissipation depends both on wave period and wave height (raised to the power of about 2.5). Thus, the calculated results have a nonlinear dependence on wave height: the computed height change between two water depths is affected by the actual value of wave height.

This nonlinear aspect implies that these computations are not exactly reversible. Projecting a wave condition offshore without dissipation from a measurement site to d_m can result in a markedly different computed dissipation rate there than if the nominally corresponding waves are projected onshore to d_m. However, the calculation procedure tends to cancel internally this effect of nonlinear height dependence.

Reversing Example Problem 2, using $H_1 = 1.69$ meters at $d_1 = 9$ meters as the initial condition, computed conditions at d_m include $\xi_m = 1.24$ meters, $f_{em} = 0.0406$ and $\overline{E}_m = 7.52$ kilograms per second cubed, but at $d_j = 5.2$ meters the calculated wave height is 1.67 meters, only 1.8 percent less than the nearshore wave height of 1.7 meters originally specified. Using computed final wave heights in Example Problem 1 as input conditions, the reverse calculation procedure gives wave heights in each case only .2 percent less than the specified height of 3.5 meters.

Such slightly irreversible results do not seem too significant for potential applications. However, the Appendix to this report provides a calculator program quickly executing the present procedure, making it convenient to examine results of reverse calculations and to determine a wave condition which appears optimally consistent with that specified.

IV. SUMMARY

The equations and procedures presented here permit calculation of nearshore wave height changes considering the energy dissipated by rough turbulent flow over a strongly agitated bed of quartz sand. All elementary wave relationships are from linear (small-amplitude) wave theory, but one effect of incorporating dissipation is that calculated height changes depend on the actual wave height. Example calculations demonstrate the conversion of a nearshore wave condition into a corresponding wave height in shallower or deeper water; the present procedures are suitable only for field waves of relatively large height and period in fairly shallow water. The general effect of energy dissipation is that nearshore wave height remains more nearly constant outside the breaker zone than linear wave theory would predict.

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APPENDIX

CALCULATOR PROGRAM FOR WAVE SHOALING WITH DISSIPATION

The following four pages document a calculator program executing the procedure presented in Section II and exemplified in Section III of this report. This program runs in about 120 seconds on a Hewlett-Packard HP-67 Programmable Pocket Calculator, employing metric units, RPN logic, three levels of nested subroutines, 17 address labels, 26 storage registers, and 223 program steps. The program could be converted for use with other calculators having different logic systems but similar features and capacities.

Equations (1) to (10) are included with an effective root-finding iteration for wavelength. Values to be specified in metric units for a calculation are: ρ, g, H_1, T_1, d_1, d_j, D_m, and X. The standard value of g is 9.81 meters per second squared, and the value of ρ for seawater may be taken as 1026 kilograms per cubic meter; for freshwater, ρ is about 1000 kilograms per cubic meter, but common situations might not constitute the requisite rough turbulent flow over a strongly agitated bed. Satisfaction of these requirements, related to equations (11) and (12), remains to be considered external to the calculator program.

Program Description

Program Title NEARSHORE WAVE SHOALING WITH ENERGY DISSIPATION

Contributor's Name ROBERT J. HALLERMEIER
Address RESEARCH DIVISION, U.S. ARMY COASTAL ENGINEERING RESEARCH CENTER
City KINGMAN BUILDING, FORT BELVOIR **State** VIRGINIA **Zip Code** 22060

Program Description, Equations, Variables FOR SPECIFIED CONDITIONS, THE PROGRAM COMPUTES REQUIRED CHARACTERISTICS OF LINEAR GRAVITY WAVES AT THREE WATER DEPTHS. THE INITIAL WAVE CONDITION AND MATERIAL CHARACTERISTICS DETERMINE EQUIVALENT WAVE HEIGHT AT THE FINAL WATER DEPTH, CONSIDERING ENERGY DISSIPATION OVER THE INTERVAL BY ROUGH TURBULENT FLOW OVER A STRONGLY AGITATED SAND BED.

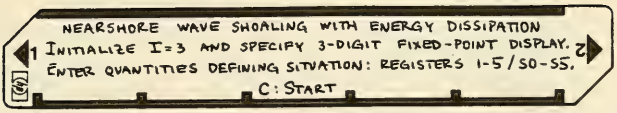
THE PROGRAM INCORPORATES EQUATIONS (1)-(10) FROM SECTION II. THE APPROPRIATE SIGN FOR EQUATION (10) IS SELECTED AUTOMATICALLY, ACCORDING TO WHETHER THE FINAL SITE IS LANDWARD OR SEAWARD OF THE INITIAL SITE. ITERATIVE SOLUTION FOR NEEDED WAVELENGTHS BEGINS WITH FIRST-GUESSES PROVIDED INTERNALLY, AND USES THE EQUATION OF THE SECANT METHOD FOR ROOT FINDING:

$$x_{i+1} = x_i - F(x_i)\left[\frac{(x_i - x_{i-1})}{F(x_i) - F(x_{i-1})}\right]$$

THESE IDENTITIES FOR HYPERBOLIC FUNCTIONS ARE USED:
$\sinh \chi = \frac{1}{2}(e^{\chi} - e^{-\chi})$; $\cosh^2 \chi = 1 + \sinh^2 \chi$; $\tanh \chi = \sinh \chi / \cosh \chi$.
NOTATION FOR VARIABLES AND CONSTANTS IS STATED ON USER INSTRUCTIONS. RESULTS FOR SEVERAL EXAMPLES ARE PROVIDED IN SECTION III OF REPORT.

Operating Limits and Warnings SEE TEXT RELATING TO EQUATIONS (11) AND (12); THE EXPRESSION USED FOR DISSIPATION COEFFICIENT IS VALID ONLY FOR HIGH-ENERGY NEARSHORE WAVES ON EXPOSED COASTS WITH RELATIVELY FINE SANDS. LINEAR MONOCHROMATIC WAVES PROPAGATING NORMAL TO SHORE WITHOUT BREAKING ARE TREATED; THIS IGNORES MANY FACTORS IN NEARSHORE WAVE TRANSFORMATION (SEE SECTION I).

User Instructions



STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1.	LOAD PROGRAM INTO CALCULATOR MEMORY AND				
	SPECIFY DISPLAY NEEDED FOR FOUR-DIGIT		f	FIX	
	ACCURACY IN ROOT-FINDING PROCEDURE.		DSP	3	
2.	INITIALIZE I-REGISTER USED FOR CONTROL		3		
	OF INTERNAL PROGRAM BRANCHING.		h	ST I	
3.	STORE QUANTITIES DEFINING SITUATION, IN				
	MKS UNITS (METERS, KILOGRAMS, SECONDS):	(MKS)			
	a. PRIMARY VARIABLES				
	INITIAL WAVE HEIGHT, SQUARED	H_1^2	STO	1	
	WAVE PERIOD	T_1	STO	2	
	WAVELENGTH IN DEEP WATER	$gT_1^2/2\pi$	STO	3	
	INITIAL WATER DEPTH MULTIPLE	$2\pi d_1$	STO	4	
	FINAL WATER DEPTH MULTIPLE	$2\pi d_j$	STO	5	
	b. SECONDARY QUANTITIES (LESS FREQUENT CHANGES)				
	EXCHANGE PRIMARY/SECONDARY STORAGE		f	P≷S	
	WAVE PROPAGATION DISTANCE	X	STO	0	
	SAND GRAIN DIAMETER MULTIPLE	$14.57\ D_m^{0.194}$	STO	1	
	NEEDED CONSTANT COEFFICIENT	5.882	STO	2	
	NEEDED CONSTANT EXPONENT	−0.194	STO	3	
	FACTOR IN ENERGY DISSIPATION RATE	$0.235\rho(2\pi)^3$	STO	4	
	FACTOR IN WAVE ENERGY FLUX	$\rho g/8$	STO	5	
	EXCHANGE PRIMARY/SECONDARY STORAGE.		f	P≷S	
4.	START PROGRAM EXECUTION.		C		(MKS)
5.	WAIT FOR STATIONARY DISPLAY OF FINAL ANSWER.				H_j
6.	EXAMINE OTHER COMPUTED QUANTITIES OF INTEREST,				
	E.G., FROM SECONDARY STORAGE REGISTERS:		f	P≷S	
	ENERGY DISSIPATION COEFFICIENT		RCL	6	f_{em}
	SPECIFIC ENERGY DISSIPATION RATE		RCL	7	$\bar{E}_m$
	SPECIFIC INITIAL WAVE ENERGY FLUX		RCL	8	$\bar{P}_1$
			f	P≷S	
7.	RETURN TO STEP 2 FOR NEW CALCULATION:				
	INITIALIZE I AND RE-ENTER ANY QUANTITIES TO				
	BE CHANGED. NOTE THAT AFTER PROGRAM RUNS				
	REGISTER 1 CONTAINS H_m^2, NOT H_1^2.				

Program Listing I

STEP	KEY ENTRY	KEY CODE	COMMENTS
00	f LBL C	31 25 13	BRANCH TO COMPUTATIONS FOR SUCCESSIVE DEPTHS; DECREMENT I- REGISTER
	GTO 3	22 03	
	f LBL 0	31 25 00	
	f DSZ	31 33	
	GTO (2)	22 24	
	h RTN	35 22	
	f LBL 3	31 25 03	INITIAL WATER DEPTH.
	RCL 4	34 04	
	STO E	33 15	
010	RCL 2	34 02	
	f GSB E	31 22 15	
	f GSB A	31 22 11	
	f GSB B	31 22 12	
	h RTN	35 22	
	f LBL 2	31 25 02	FINAL WATER DEPTH.
	RCL 5	34 05	
	STO E	33 15	
	RCL 2	34 02	
	f GSB E	31 22 15	
020	f GSB A	31 22 11	
	h RTN	35 22	
	f LBL 1	31 25 01	INTERMEDIATE DEPTH: EQUATION (6)
	RCL 4	34 04	
	RCL 5	34 05	
	x	71	
	f $\sqrt{x}$	31 54	
	STO E	33 15	
	RCL 0	34 00	
	f GSB E	31 22 15	
030	g GSB fa	32 22 11	
	h RTN	35 22	
	f LBL E	31 25 15	ITERATIVE ROOT FINDER USING SECANT METHOD.
	f GSB D	31 22 14	
	RCL 8	34 12	
	GTO 4	22 04	
	f LBL 6	31 25 06	
	RCL 0	34 00	
	f GSB 5	31 22 05	
	STO B	33 12	
040	f LBL 4	31 25 04	
	RCL A	34 11	
	RCL 0	34 00	
	STO A	33 11	
	−	51	
	RCL D	34 14	
	RCL B	34 12	
	STO D	33 14	
	−	51	
	÷	81	
050	x	71	
	STO − 0	33 51 00	
	RCL 0	34 00	
	÷	81	
	f RND	31 24	
	f x≠0	31 61	
	GTO 6	22 06	

STEP	KEY ENTRY	KEY CODE	COMMENTS
	RCL 0	34 00	
	h RTN	35 22	
	f LBL D	31 25 14	FINITE-DIFFERENCE APPROXIMATION FOR FIRST DERIVATIVE IN SECANT METHOD.
060	EEX	43	
	CHS	42	
	2	02	
	f %	31 82	
	STO C	33 13	
	2	02	
	÷	81	
	−	51	
	STO A	33 11	
	STO 0	33 00	
070	f GSB 5	31 22 05	
	STO D	33 14	
	RCL A	34 11	
	RCL C	34 13	
	+	61	
	STO 0	33 00	
	f GSB 5	31 22 05	
	STO B	33 12	
	RCL D	34 14	
	−	51	
080	RCL C	34 13	
	÷	81	
	h RTN	35 22	
	f LBL 5	31 25 05	L DEFINITION: EQUATION (1).
	RCL E	34 15	
	RCL 0	34 00	
	÷	81	
	g e^x	32 52	
	ENTER ↑	41	
	h $1/x$	35 62	
090	−	51	
	2	02	
	÷	81	
	STO 6	33 06	
	g x^2	32 54	
	g_1	01	
	+	61	
	f $\sqrt{x}$	31 54	
	STO 7	33 07	
	RCL 6	34 06	
100	÷	81	
	h $1/x$	35 62	
	RCL 0	34 00	
	÷	81	
	RCL E	34 15	
	x	71	
	RCL 3	34 03	
	h $1/x$	35 62	
	RCL E	34 15	
	x	71	
110	−	51	
	h RTN	35 22	
	R/S	84	

REGISTERS

0 $L_i; L_i; L_i; L_i$	1 $H_i^2; H_6^2; H_m^2$	2 T_i	3 $gT_i^2/2\pi$	4 $2\pi d_i$	5 $2\pi d_i$	6 sinh	7 cosh	8 $m_i; m_j; m_m$	9 Σm
S0 X	S1 14.57 $D_m^{0.94}$	S2 5.882	S3 −0.194	S4 $0.235p(2\pi)^3$	S5 $pg/8$	S6 f_{em}	S7 $\overline{E}_m$	S8 $\overline{P}_i$	S9 (cm), (cm);
A L_{i-1}	B $F(L_i)$	C ΔL	D $F(L_{i-1})$	E $2\pi d_i; 2\pi d_j; 2\pi\sqrt{dd_i}$	I INDEX				

Program Listing II

STEP	KEY ENTRY	KEY CODE	COMMENTS
	f LBL A	31 25 11	Computation of (mc).
	f GSB 7	31 22 07	
	RCL 8	34 08	
	RCL 0	34 00	
	x	71	
	RCL 2	34 02	
	÷	81	
120	f P≥S	31 42	
	STO 9	33 09	
	f P≥S	31 42	
	h RC I	35 34	
	2	02	
	−	51	
	f x=0	31 51	
	GTO 0	22 00	
	h RTN	35 22	
	f LBL 7	31 25 07	Computation of m: Equation (3).
130	RCL 6	34 06	
	RCL 7	34 07	
	x	71	
	RCL 0	34 00	
	x	71	
	h √x	35 62	
	RCL E	34 15	
	x	71	
	1	01	
	+	61	
140	2	02	
	÷	81	
	STO 8	33 08	
	h RTN	35 22	
	f LBL B	31 25 12	Computation of $\bar{P}_i, K_{Si}^2$: Equations (2), (4).
	RCL 1	34 01	
	f P≥S	31 42	
	RCL 9	34 09	
	x	71	
	RCL 5	34 05	
150	x	71	
	STO 8	33 08	
	f P≥S	31 42	
	RCL 6	34 06	
	RCL 7	34 07	
	÷	81	
	RCL 8	34 08	
	x	71	
	2	02	
	x	71	
160	STO x 1	33 71 01	
	GTO 0	22 00	
	h RTN	35 22	
	g LBL Fa	32 25 71	Computations for intermediate depth: Equations (5), (7)-(10), with sign choice.
	g f GSB 7	31 22 07	
	RCL 8	34 08	
	2	02	
	x	71	
	RCL 6	34 06	

STEP	KEY ENTRY	KEY CODE	COMMENTS
	x	71	
170	RCL 7	34 07	
	÷	81	
	STO ÷ 1	33 81 01	
	RCL 1	34 01	
	f √x	31 54	
	2	02	
	÷	81	
	RCL 6	34 06	
	÷	81	
	STO 9	33 09	
180	f P≥S	31 42	
	RCL 3	34 03	
	h y^x	35 63	
	RCL 1	34 01	
	x	71	
	RCL 2	34 02	
	−	51	
	g e^x	32 52	
	STO 6	33 06	
	RCL 4	34 04	
190	x	71	
	f P≥S	31 42	
	RCL 9	34 09	
	RCL 2	34 02	
	÷	81	
	3	03	
	h y^x	35 63	
	x	71	
	f P≥S	31 42	
	STO 7	33 07	
200	f P≥S	31 42	
	RCL 5	34 05	
	RCL 4	34 04	
	f P≥S	31 42	
	g x>y	32 81	
	GTO 8	22 08	
	GTO 9	22 09	
	f LBL 8	31 25 08	
	RCL 7	34 07	
	CHS	42	
210	GTO Fe	22 31 15	
	f LBL 9	31 25 09	
	RCL 7	34 07	
	g LBL Fe	32 25 15	
	g RCL 0	34 00	
	x	71	
	RCL 8	34 08	
	+	61	
	RCL 5	34 05	
	÷	81	
220	RCL 9	34 09	
	÷	81	
	f √x	31 54	
	f P≥S	31 42	
	h RTN	35 22	

LABELS

A mc EQN.	B $\bar{P}_i, K_{Si}$ EQNS.	C START	D DIFFERENTIAL	E ROOT FINDER
a d_m CALCS.	b ——	c ——	d ——	e H_i EQN.
0 I DECREMENT	1 d_m BRANCH	2 d_i BRANCH	3 d_i BRANCH	4 USED
5 L EQN.	6 ITERATION	7 m EQN.	8 SIGN CHANGE	9 USED

FLAGS

0	——
1	——
2	——
3	——

SET STATUS

FLAGS	ON	OFF
0	☐	☒
1	☐	☒
2	☐	☒
3	☐	☒

TRIG	
DEG	☒
GRAD	☐
RAD	☐

DISP	
FIX	☒
SCI	☐
ENG	☐
n	3

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